Study of anomalous kinetics of the annihilation reaction $A + A \to \varnothing$ in the framework of an effective field-theoretic model.

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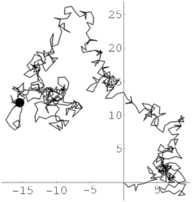

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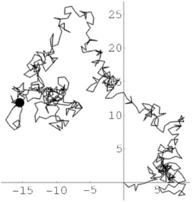

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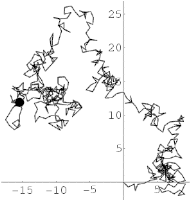

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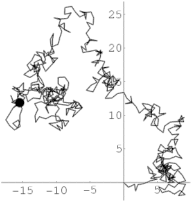

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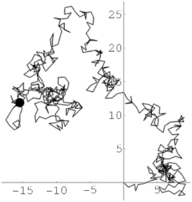

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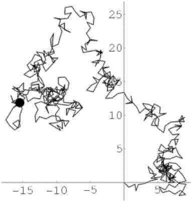

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- first case ⇒ classical rate eq. $\frac{dn(t)}{dt} = -kn^2(t) \rightarrow n(t) \propto t^{-1}$

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- Lee, J. Phys. A **27**, 2633 (1994); Peliti, J. Phys. A **19**, L365 (1986)

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$$\frac{\partial}{\partial t} P_N(q^N, t) + H(t)\, P_N(q^N, t) = 0, \tag{1}$$

where $q^N = (q_1, q_2, \dots, q_N)$ and q_i is whole set of coordinates (generalized momentum, coordinate etc.)

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- $n(\mathbf{x},t) = \langle 0|e^{\int d\mathbf{x}\psi}\psi^\dagger(\mathbf{x})\psi(\mathbf{x})|\Phi(t)\rangle = \langle 0|\psi(\mathbf{x})e^{-\hat{H}(\psi^\dagger+1,\psi)}e^{\int d\mathbf{x}\psi}|\Phi(0)\rangle$

Casting into the path integral representation

- formal solution for $|\Phi(t)\rangle$ and after some steps

$$\begin{aligned}\langle A(t)\rangle &= \langle 0|TA\{[\psi^{+}(t)+1]\psi(t)\}) \\ &\quad \exp(-\int_0^\infty \hat{H}_I' dt + n_0 \int d\mathbf{x}\psi^{+}(\mathbf{x},0))|0\rangle \end{aligned} \tag{16}$$

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- expectation value $\rightarrow$ path integral representation [A. N. Vasiliev, *Functional Methods in Quantum Field Theory and Statistical Physics*]

$$\langle A(t)\rangle = \int \mathcal{D}\psi^+\mathcal{D}\psi A\{[\psi^+(t)+1]\psi(t)\}\mathrm{e}^{\mathrm{S_{react}}} \tag{17}$$

Casting into the path integral representation

- formal solution for $|\Phi(t)\rangle$ and after some steps

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$$\begin{aligned} S_{react} = -\int_0^\infty dt \int d\mathbf{x}\, \{\psi^+\partial_t\psi + \psi^+\nabla(\mathbf{v}\psi) - D_0\psi^+\nabla^2\psi + \\ \lambda_0 D_0[2\psi^+ + (\psi^+)^2]\psi^2\} + n_0\int d\mathbf{x}\,\psi^+(\mathbf{x},0) \end{aligned} \tag{18}$$

Environment

How to describe advecting environment?

- Navier-Stokes eq.

$$\partial_t \mathbf{v} + (\mathbf{v}.\nabla)\mathbf{v} = \nu_0 \nabla^2 \mathbf{v} - \frac{\nabla p}{\rho} + \mathbf{f}^v \qquad (19)$$

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$$S_{NS} = \frac{1}{2} \int dt\, d\mathbf{x}\, d\mathbf{x}'\, \tilde{\mathbf{v}}(\mathbf{x}, t).\tilde{\mathbf{v}}(\mathbf{x}', t) d_f(|\mathbf{x} - \mathbf{x}'|) + \int dt\, d\mathbf{x}\, \tilde{\mathbf{v}}.[-\partial_t \mathbf{v} - (\mathbf{v}.\nabla)\mathbf{v} + \nu_0 \nabla^2 \mathbf{v}] \quad (20)$$

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- any statistical quantity with respect to the concentration and velocity fluctuations could now be averaged with the use of weight functional

$$\mathcal{W} = \mathrm{e}^{S_{react} + S_{NS}} \tag{21}$$

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- $\Gamma_{\psi^+\psi v}$ and $\Gamma_{vvv'}$ convergent because of Galilei invariance [L. Ts. Adzhemyan, A. N. Vasiliev, Yu. M .Pis'mak, Teor. Mat. Fiz. **57**, 268 (1983)]

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- RG functions

$$\beta_g = \mu \frac{\partial g}{\partial \mu}|_0, \gamma_\alpha = \mu \frac{\partial \ln Z_\alpha}{\partial \mu}|_0 \qquad (24)$$

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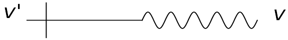
- where

$$\alpha_{R1} = g_1\overline{S_d}s^{2\epsilon}Z_1^{-3}$$
$$\alpha_{R2} = g_2\overline{S_d}s^{-2\Delta}Z_3Z_1^{-3}$$
$$\alpha_{R3} = \lambda\overline{S_d}s^{-2\Delta}Z_2^{-1}Z_4$$
$$s = \mu/p$$

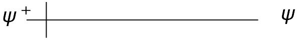
Definitions of propagators and vertex factors

from the actions, (18) and (20), follows

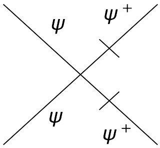
$<v\, v>_0 = d_f(k)/(\omega^2 + \nu_0 k^4)$



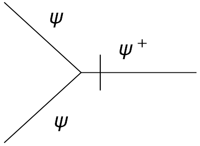
$<v'\, v>_0 = 1/(-i\omega + \nu_0 k^2)$

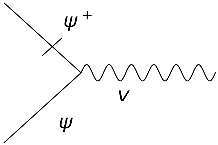


$<\psi^+\, \psi>_0 = 1/(-i\omega + D_0 k^\wedge 2)$



$4\lambda_0\, D_0$





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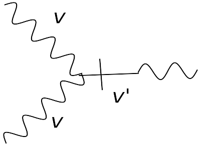


$V_{ijs} = i(\delta_{ij} k_s + \delta_{is} k_j)$

Results of one-loop order

- beta functions could be obtaind directly from definition

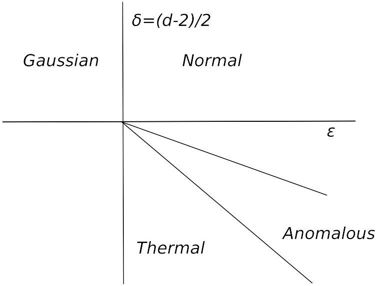
$$
\begin{aligned}
\beta_{g_1} &= g_1(-2\epsilon + 3\gamma_1), \beta_{g2} = (2\delta + 3\gamma_1 - \gamma_3) \\
\beta_\lambda &= \lambda(2\delta - \gamma_4 + \gamma_2), \beta_u = u(\gamma_1 - \gamma_2)
\end{aligned} \tag{27}
$$

- and gamma functions in the explicit form (from the knowledge of $Z_i, i = 1, 2, 3, 4$)

$$
\begin{aligned}
\gamma_1 &= \frac{g_1 + g_2}{32\pi}, \gamma_2 = \frac{g_1 + g_2}{8\pi u(1+u)} \\
\gamma_3 &= \frac{(g_1 + g_2)^2}{32\pi g_2}, \gamma_4 = -\frac{\lambda}{2\pi}
\end{aligned} \tag{28}
$$

Fixed points of RG group

density decay rate $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability

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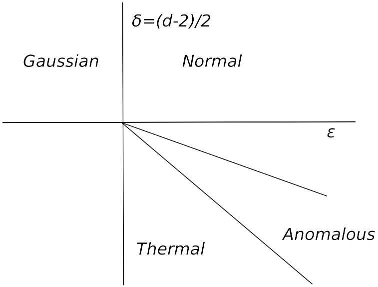

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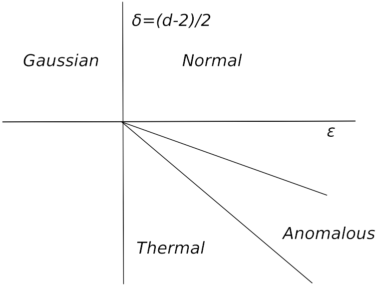

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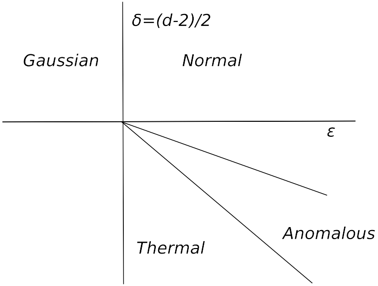

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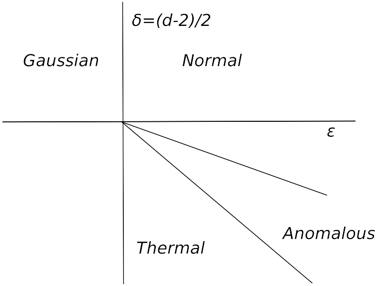

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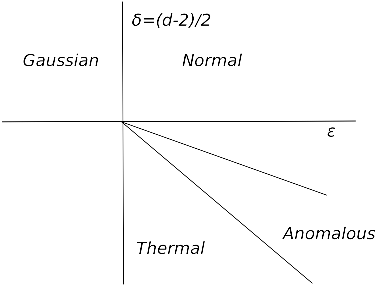

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Driftless	$1 + \delta$	unstable

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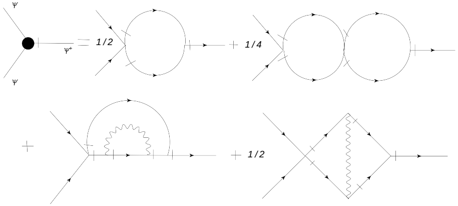
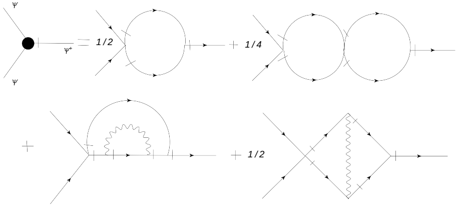
- what will be done in the nearest future
  (a) two-loop calculation almost completed



Conclusions

- field-theoretic model for the one-species annihilation reaction was constructed
- fixed points of RG were found in 1-loop order
- scaling dimension α for decay of concetration was calculated at tree level
- what will be done in the nearest future
  (a) two-loop calculation almost completed



  (b) adding random source and sinks of *A* particles

Thank you for your attention